A Trigonometry Mistake

New Way Additional Mathematics Book 1 p.227

Example 27

Solve the equation $\tan 3x = \cot 2x$

Proposed Solution

$$\begin{aligned} \tan 3x &= \cot 2x \\ \tan 3x &= \tan (90^\circ - 2x) \\ 3x &= 180^\circ n + (90^\circ - 2x) \\ 5x &= 180^\circ n + 90^\circ \\ x &= 36^\circ n + 18^\circ, \quad \text{where } n \text{ is an integer .} \end{aligned}$$

Analysis

Some of the solution can make $\tan 3x$ or $\cot 2x$ tend to infinity . It is necessary to reject those roots which are not real numbers.

Note that the roots within the range $0^\circ \le x < 360^\circ$ are

$18^\circ, 54^\circ, 90^\circ, 126^\circ, 162^\circ, 198^\circ, 234^\circ, 270^\circ, 306^\circ, 342^\circ$ …. (1)

It is easy to check that the other solutions are periodic. The general solutions are of the form :

$18^\circ + 360^\circ n$, $54^\circ + 360^\circ n$, $90^\circ + 360^\circ n$, $126^\circ + 360^\circ n$, $162^\circ + 360^\circ n$,
$198^\circ + 360^\circ n$, $234^\circ + 360^\circ n$, $270^\circ + 360^\circ n$, $306^\circ + 360^\circ n$, $342^\circ + 360^\circ n$ …. (2)

However,

If $x = 90^\circ + 360^\circ n$, $\tan 3x = \tan 3(90^\circ + 360^\circ n) = \tan 270^\circ = \infty$
$\cot 2x = \cot 2(90^\circ + 360^\circ n) = \cot 180^\circ = \infty$

If $x = 270^\circ + 360^\circ n$, $\tan 3x = \tan 3(270^\circ + 360^\circ n) = \tan 810^\circ = \tan 90^\circ = \infty$
$\cot 2x = \cot 2(270^\circ + 360^\circ n) = \cot 540^\circ = \cot 180^\circ = \infty$

Therefore these solutions should be rejected.

The complete solution for $\tan 3x = \cot 2x$ are

$18^\circ + 360^\circ n$, $54^\circ + 360^\circ n$, $126^\circ + 360^\circ n$, $162^\circ + 360^\circ n$,
$198^\circ + 360^\circ n$, $234^\circ + 360^\circ n$, $306^\circ + 360^\circ n$, $342^\circ + 360^\circ n$, where n is an integer .